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Seasonal Variation of Density and Temperature over Churchill, Canada, during Solar Maximum

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Abstract. Atmospheric density and temperature data obtained through application of a technique employing radioactive ionization-gage nose-cone surface-pressure measurements have indicated a seasonal variation of these parameters in the auroral zone at a time of maximum solar activity (1958). A density increase by a factor of 2 to 3 in the summer, centered in the 70- to 80-km region, is observed. Correspondingly, the temperature in the same region decreases and shows much less fine structure than for winter conditions. Density data from nine flights and temperature data for five flights are presented. A previously unpublished temperature profile for White Sands, New Mexico, for the previous solar minimum is presented for comparison.

Introduction. The IGY Rocket Program conducted at Fort Churchill was devoted to the determination of many properties of the upper atmosphere, including the structural parameters ambient pressure, temperature, density, and winds. Of a total of about ninety [*Siewert*, 1959] rockets launched for all probing purposes, 31 were used partly or completely to make these structural measurements, which were carried out during all seasons of the year and at all times of the day, insofar as practical considerations, such as weather and the realities of rocket launchings, would permit. One series of experiments, with the determination of these structural parameters as the objective, employed the 'grenade' technique [*Stroud et al.*, 1960]; another used the 'falling sphere' technique [*Jones et al.*, 1959]; a third, the pitot-static tube technique, concerned with measurements above 100 km as well as below, employed various types of pressure measurement devices in special geometric configurations [*Ainsworth et al.*, 1961]. The fourth technique, called the pressure gage experiment, enabled determination of the desired quantities from basic conical nose-cone surface-pressure measurements. This paper is a report of the series of measurement that used the fourth technique.

Plan of the measurement program. Several Aerobee rocket flights employing radioactive ionization gages for nose-cone surface-pressure measurement had been carried out during the years before initiation of the IGY program [*Sicinski et al.*, 1954]; these experiments provided data yielding values of atmospheric pressure, temperature, and density to altitudes of approximately 80 km. In addition to giving useful scientific information, the flights provided experience with techniques which were adaptable to a concentrated series of measurements appropriate to the IGY effort. Before the IGY only about five launchings were carried out; during IGY sixteen launchings were planned for the world effort to enable a frequency of measurements which would provide data relative to seasonal, as well as diurnal, variation of the desired parameters. Three of these were Aerobee launchings. The less expensive Nike-Cajun rocket, which carried less complex instrumentation, was chosen for the rest of the launchings. The magnitude of the anticipated program required simplification of the instrumentation that had been employed for the early flights.

Of the originally planned sixteen launchings, ten were ultimately carried out, with partial success both in data and seasonal coverage (see Table 1). Three additional launchings, devoted to the measurement of other parameters as their primary objective, provided some further atmospheric structure data. Three versions of the

TABLE 1. Flight Information

Flight Serial No.	Rocket	Date	Time, CST	Data Return	Pressure Measurements
AM 6.31	NC	10/20/56	1601	Poor	ram + 2 cone wall
AM 2.21	A	10/23/56	0240	Good	ram + 4 cone wall
AM 6.32	NC	7/30/57	1210	None (No second stage ignition)	ram + 2 cone wall
AM 4.01	A	9/1/57	1628	Limited altitude range (Partial instrumentation failure)	ram + 4 conewall
AM 6.34	NC	12/14/57	1837	None (Rocket structural failure, instrumentation recovered, reflown on AM 6.37)	ram + 2 cone wall
AM 6.36	NC	1/27/58	1317	Poor (Very high yaw angles)	ram + 2 cone wall
AM 6.37	NC	2/24/58	0134	Good	ram + 2 cone wall
AM 6.38	NC	3/24/58	1600	Good	ram + 2 cone wall
AM 6.39	NC	7/15/58	1407	Good	ram + 2 cone wall
AM 4.12	A	10/14/58	1904	Good	ram + 4 cone wall
ABM 6.207	NC	10/20/58	1601	Good	ram pressure only (secondary experiment)
AA 6.16	NC	11/23/58	1602	Good	ram pressure only (secondary experiment)
AA 6.18	NC	11/—/58		None (Nose cone failure of primary experiment)	ram pressure only (secondary experiment)

basic experimental technique were employed in these thirteen launchings. These will be described below.

Basis for the experiments. A detailed determination of the ambient pressure, temperature, and density of the atmosphere during the flight of a rocket is possible if we can obtain sufficiently precise measurements of (*a*) pressures at prescribed surface locations on a right circular cone (the rocket's nose cone), (*b*) the velocity of the cone with respect to the stream, (*c*) the angle of attack of the cone with respect to the stream (to make analysis feasible it must always be less than 20° to 25°), and (*d*) the altitude versus time history of the flight.

Generally speaking, the more detailed information we can get regarding the pressure distribution about the nose cone, the more precise information we can deduce regarding the undisturbed air just ahead of the rocket. When the rocket has zero angle of attack, that is, when the stream velocity vector is aligned with the geometric axis of the rocket, the pressure distribution relationships are quite well known and not complex, and thus ambient data can be quite readily determined. When the angle of attack is not zero, but is some small angle of the order of, or less than, the half-angle of the cone, the relationships are somewhat complex and not entirely well defined. Under these conditions data analysis can be successfully carried out, but it becomes a rather lengthy and involved procedure, even assuming that we have knowledge of the rocket angle of attack with a precision of better than one degree. For a thorough analysis of data obtained for angles greater than the cone half-angle, the process becomes prohibitively complex. Application of machine computation techniques helps, but the flow theory has not been adequately defined to permit proper analysis.

The general procedure involves computation of the free stream Mach number and ambient pressure from the measured raw and non-yawed surface pressure [*Sicinski et al.*, 1954]. Subsequently, ambient temperature and density are computed from the Mach number and ambient pressure (see appendix for details). Alternatively, or for comparative data, the ram pressure and rocket velocity considered together enable a measurement of ambient density (see appendix). Application of the hydrostatic equation then permits a temperature determination.

Thus, a complete instrumentation makes possible the application of several related techniques for data analysis. As was noted previously, however, modest angles of attack make it necessary to employ a considerably more complex data analysis technique involving the correction of 'yawed' cone-surface pressure to the value that would have been observed in the absence of any angle of attack. The ram-pressure measurement, on the other hand, is relatively insensitive to the angle of attack of the nose cone for angles as large as about 20°, for selected sampling-port configurations. Thus, launchings which employ cone-pressure measurement techniques possess significant potential for extensively detailed information, being able to provide a large number of data points per kilometer of altitude. At the same time, the experiment is relatively sensitive to attitude perturbation, which can seriously restrict the potential of the experiment, as will be shown.

Practical aspects of the measurements. Three variations of the basic experiment were employed for the IGY launchings. The 'Aerobee' form of the experiment employed the relatively complete and somewhat redundant implementation of the experiment wherein an effort was made to determine the pressure characteristics of the flow about the nose cone, and the angle of attack of the nose cone to the stream. In this form of the experiment, five pressures were measured; one at the nose-cone apex (called the total head or ram pressure), and four at equally spaced points on the cone surface, well downstream of the ram-pressure port. In addition, a gyro was included to provide data to assist in determination of the angle of attack.

In a less complex version of the experiment that was employed for most Nike-Cajun launchings, only three pressures were measured, including the ram pressure and the surface pressure at two diametrically opposite cone wall points. For this experiment, the gyro was omitted, for it was expected that 'non-yawed' pressure values would be deduced from the yawed cone wall pressure values, using a technique that amounts to an averaging process (low angles of attack).

The least complex version of the experiment, used for only two Nike-Cajun flights, employed only a single pressure measurement system to determine the ram (total head) pressure, an arrangement relatively insensitive to angle of attack, as we discussed.

The altitude range over which measurements can be made by the techniques discussed is determined by several factors. The lower limit, around 30–40 km, was chosen on the basis of check points available at that altitude from balloon measurements. The upper limit, which for the measurements reported in this paper increased from about 75 to 100 km as the technique was refined, is imposed basically by a decreasing Reynolds number, an increasing mean free path, and the changing composition of the atmosphere. The expected ambient pressure range corresponding to the 30- to 100-km altitude range and the expected nose-cone velocity range (3000–6000 fps) imposed a pressure measurement range requirement of several decades. For example, at 30 km the ambient pressure is approximately 10 mm Hg and the corresponding ram pressure approximately 25 times this pressure, thereby requiring that the pressure measurement system respond to a maximum pressure of 250 mm Hg. At the upper-altitude limit, the ambient pressure is approximately 10^{-3} mm Hg, and, since this value can be considered the minimum pressure which the cone wall instruments could experience, the lower limit imposed on a pressure measurement device to be used for this experiment was defined.

The pressure-sensor devices used for these measurements employed radioactive ionization gages and unity feedback DC amplifiers with automatic range changing arrangements. These systems have been reported by *Spencer et al.* [1959].

The pressure gage calibration system employed consists of a glass manifold connected at one end through a gate valve to the pumping equipment. Attached to the other end was a second gate valve, and a short metal manifold, used for mounting one to three units, thus permitting simultaneous calibrations. The pumping section was composed of a water-cooled, three-stage fractionating pump with the usual mechanical fore pumps. With this combination, pressures of 10^{-6} mm Hg, well below the required minimums, were easily attained. A McLeod gage (CVC type MG-07) was used as an absolute standard for measuring pressures from 10^{-4} to 5 mm Hg. An error of less than

2 per cent for pressures above 10^{-3} mm Hg can be realized with the instrument. For higher pressures, aneroid gages with an accuracy of better than 2 per cent were used.

The vacuum system was also equipped with the usual accessories for system protection in case of glass breakage, for diffusion pump control, and for drying air. Liquid nitrogen traps were installed between the McLeod gage and the system to prevent mercury vapor from entering the system.

During normal usages the presence of leaks and outgassing was checked by pumping the system to a low pressure (about 10^{-6} mm Hg), closing the gate valve to the pump, and then observing the rate of increase in pressure. Without bake-out of the flight gages, not feasible or necessary in this case, a small increase due to outgassing was observed. It was required only that the pressure not rise above about 10^{-4} mm Hg in a five-minute interval (with pumps shut down) before considering the system ready for calibration. This rate of outgassing, however, is practically undetectable and thus negligible for pressures above about 10^{-3} mm Hg, the lowest pressure considered for flight.

For calibration, the following procedure was adopted. The system and units were pumped and maintained at 10^{-6} mm Hg for 48 hours. The pressure was then increased (introduction of dry room air) at intervals and the system allowed to come to equilibrium (about one minute) before recording data. The mercury level in the McLeod gages was then raised just enough to seal the measuring volume, at which time the output and reference voltages were read. After these values were recorded, the mercury in the McLeod gage was raised further to determine the pressure. This procedure was repeated at about ½-volt intervals (output range 0–5 volts) always increasing pressure. When the maximum pressure voltage (0 volts) on each range was reached, the output values for the next higher pressure range of the ionization gage system were then read for the same value of pressure. This cross-check between

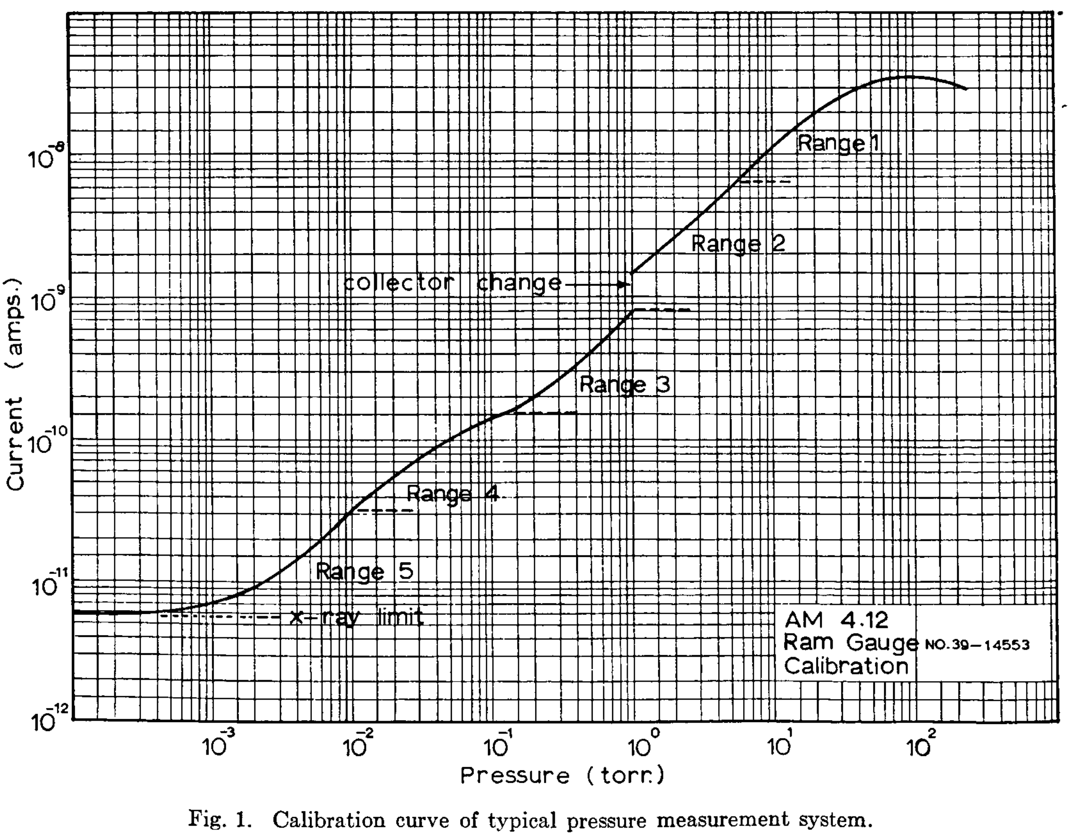

Fig. 1. Calibration curve of typical pressure measurement system.

ranges permits us to compute, and therefore check, the ratio of values of the hi-meg resistors between two consecutive ranges. This ratio is also useful in determining the true current-pressure characteristic of the ionization gage.

After the calibration was performed through all the ranges, as described above, the system was again pumped down and allowed to outgas for another 48 hours. A check calibration was then performed, during which two points per range were checked. If these points were within 2 per cent of the previous data, the calibration was considered satisfactory. A representative calibration curve is shown in Figure 1.

Data obtained. Density data, considered typical of that resulting from a fully successful flight, are shown in Figure 2. Very little smoothing has been done, the lack of scatter reflecting the quality of the raw telemetry data. This data from flight serial AM 4.12 and that resulting from other flights is shown in Table 2.

As we noted in the introduction, one of the objectives of these experiments was to provide data that would permit determination of a possible seasonal variation of the structural parameters. To this end, selected values of density data from all radioactive-ionization-gage experiments are plotted in Figure 3, which is a composite of seven plots of density versus calendar date. Altitude intervals of 10 km have been chosen to illustrate the increasing seasonal variation with altitude.

Figure 4 is a similar plot to which has been added Churchill IGY data obtained from other structural-parameter flight experiments employing grenades, falling sphere, and pitot-static tube, thus providing a plot of all comparable density data measured at Ft. Churchill by rockets during the IGY. A period of approximately two years, generally concurrent with solar maximum, from October 1956 to November 1958, is represented by these data.

Figure 5 presents three temperature profiles: the single temperature curve obtained during the 1958 summer, with the 1961 CIRA reference atmosphere plotted for comparison, and an October 1952 temperature profile measured at White Sands, New Mexico (not previously published), also presented for comparison. The latter was determined using the Mach-number data-analysis technique discussed previously.

Figure 6 illustrates Churchill winter tempera-

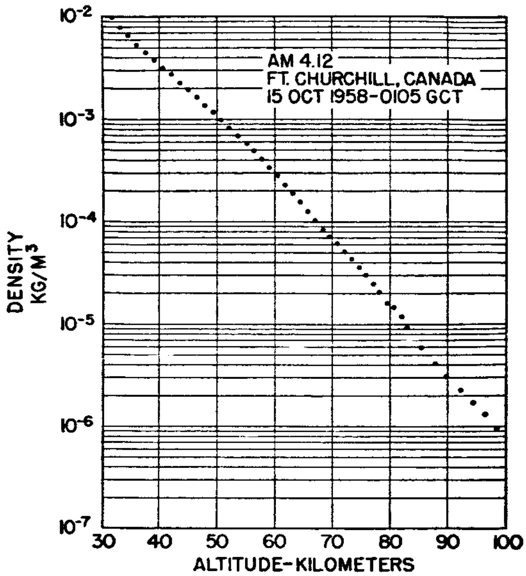

Fig. 2. Plot of density versus altitude for flight AM 4.12.

tures from four flights. An 'eyeball' average is drawn, and the detailed temperature structure from AM 4.12 is shown by a point-connecting curve to emphasize the 'fine' structure in temperature. As in Figure 5, the 1961 reference atmosphere and the 1952 White Sands measurement are shown for comparison. Two balloon reference points, one for 1952 and one for 1956, have also been included.

Discussion of the experiments and data. As we discussed previously, the experiments produced some redundancy in that atmospheric structure data could be determined through computation of the Mach number of the flow about the nose cone from ram and corrected cone wall pressures or, alternatively, from ram pressure alone. Although a rather considerable effort has been applied to analysis of the data from the flights using the Mach-number technique, it has been concluded reluctantly that (*a*) the errors encountered due to excessive rocket angle of attack, (*b*) the excessive errors encountered in its measurement, and (*c*) the insufficient precision and resolution in the ram-pressure data do not permit adequate confidence in enough of the data resulting from such an analysis to warrant their publication. On the other hand, analysis of ram-pressure data has provided valid information which we believe

TABLE 2. Density Versus Altitude

AM 2.21 10/23/56 Altitude (km)	Density (kg/m^3)
31.35	$1.49(10^{-2})$
32.72	1.23
34.08	1.00
35.43	$8.29(10^{-3})$
36.77	6.79
38.10	5.63
39.41	4.61
40.72	3.81
42.02	3.11
43.31	2.56
44.59	2.10
45.86	1.76
47.12	1.48
48.37	1.26
49.60	1.06
50.84	$8.98(10^{-4})$
52.05	7.59
53.27	6.44
54.47	5.55
55.66	4.71
56.84	3.92
58.01	3.36
59.17	2.87
60.32	2.43
61.47	2.08
62.60	1.78

AM 4.01 9/1/57 Altitude (km)	Density (kg/m^3)
60.47	$3.80(10^{-4})$
61.83	3.14
63.18	2.64
64.53	2.16
65.86	1.78
67.18	1.44
68.50	1.17
69.80	$9.34(10^{-5})$
71.09	7.42

AM 6.36 1/29/58 Altitude (km)	Density (kg/m^3)
57.5	$3.20(10^{-4})$
59.0	2.70
61.5	2.00
62.5	1.50
63.5	1.40
64.5	1.24
65.5	1.20
67.5	$8.5(10^{-5})$
68.0	7.7
69.5	6.1

AM 6.37 2/24/58 Altitude (km)	Density (kg/m^3)
53.33	$5.92(10^{-4})$
54.64	5.05
55.95	4.44
57.25	3.71
58.54	3.17
59.82	2.73
61.09	2.34
62.35	2.03
63.60	1.73
64.84	1.53
66.07	1.32
67.29	1.14
68.50	1.99
69.70	$8.75(10^{-5})$
70.89	7.69
72.07	6.74
73.25	5.99
74.41	5.26
75.56	4.70
76.70	4.15
77.84	3.71
78.96	3.31
80.08	2.95
81.18	2.67
82.28	2.36
83.36	2.12
84.44	1.91
85.51	1.70
86.57	1.51
87.61	1.44
88.65	1.40
89.68	1.31

AM 6.38 3/24/58 Altitude (km)	Density (kg/m^3)
58.57	$2.62(10^{-4})$
59.81	2.23
61.04	1.94
62.27	1.65
63.48	1.41
64.69	1.23
65.88	1.06
67.06	$9.21(10^{-5})$
68.24	8.00
69.40	6.99
70.56	6.22
71.70	5.35
72.84	4.68
73.97	4.11
75.09	3.60
76.19	3.15
77.29	2.75
78.38	2.42
79.46	2.11
80.53	1.87
81.59	1.67
82.64	1.49
83.67	1.32
84.71	1.19
85.73	1.07
86.74	$9.63(10^{-6})$
87.74	8.81
88.73	7.99
89.72	7.26
90.69	6.63

AM 6.39 7/15/58 Altitude (km)	Density (kg/m^3)
38.71	$7.18(10^{-3})$
40.01	5.63
41.29	4.54
42.57	3.70
43.83	3.11
45.09	2.59
46.33	2.19
47.56	1.88
48.78	1.61
49.99	1.38
51.20	1.18
52.39	1.02
53.57	$8.90(10^{-4})$
54.74	7.76
55.90	6.72
57.05	5.91
58.20	5.19
59.33	4.50
60.45	3.91
61.56	3.45
62.67	2.99
63.76	2.57
64.84	2.23
65.91	1.92
66.98	1.65
68.03	1.50
69.07	1.34
70.11	1.18
71.13	1.03
72.15	$9.10(10^{-5})$
73.15	7.90
74.15	6.79
75.13	5.79
76.11	$5.20(10^{-5})$
77.07	4.53
78.03	3.94
78.98	3.45
79.91	3.00
80.84	2.63
81.76	2.28
82.67	2.04
83.57	1.73
84.46	1.51
85.34	1.32

AM 4.12 10/14/58 Altitude (km)	Density (kg/m^3)
31.75	$9.81(10^{-3})$
33.25	8.00
34.75	6.45
36.26	5.37
37.76	4.50
39.25	3.79
40.74	3.19
42.21	2.71
43.67	2.28
45.13	1.93
46.58	1.63
48.01	1.37
49.44	$1.17(10^{-3})$
50.86	$9.66(10^{-4})$
52.27	8.08
53.66	6.83
55.05	5.75
56.43	4.82
57.80	4.03
59.16	3.35
60.51	2.77
61.85	2.27
63.18	1.89
64.51	1.55
65.82	1.26
67.12	$1.02(10^{-4})$
68.41	$8.40(10^{-5})$
69.69	7.06
70.97	6.05
72.23	5.14
73.49	4.35
74.73	3.57
75.96	2.95
77.19	$2.43(10^{-5})$
78.41	2.02
79.61	1.69
80.81	1.43
82.00	1.19
83.17	$9.1(10^{-6})$
85.50	5.9
87.79	4.15
90.04	3.01
92.25	2.25
94.42	1.70
96.56	1.30
98.66	$9.6(10^{-7})$

ABM 6.207 10/20/58 Altitude (km)	Density (kg/m^3)
52.62	
53.84	$6.45(10^{-4})$
55.04	5.49
56.24	4.50
57.24	3.74
58.60	3.15
59.76	2.62
60.92	2.13
62.06	1.80
63.20	1.50
64.33	1.28
65.44	$1.09(10^{-4})$
66.55	$9.30(10^{-5})$
67.64	8.08
68.73	6.90
69.81	5.99
70.88	5.26
71.93	$4.41(10^{-5})$

AA 6.16 11/23/58 Altitude (km)	Density (kg/m^3)
55.03	$4.97(10^{-4})$
56.23	4.19
57.42	3.55
58.59	3.03
59.76	2.57
60.92	2.21
62.06	1.90
63.20	1.64
64.33	1.43
65.45	1.23
66.56	$1.08(10^{-4})$
67.60	$9.33(10^{-5})$
68.75	8.06
69.83	6.93
70.90	5.77
71.96	4.67
73.01	3.76
74.05	3.08
75.09	2.61
76.11	2.32
77.12	2.11
78.13	1.87
80.10	1.25
81.09	$1.09(10^{-5})$
82.04	$9.86(10^{-6})$
83.00	8.91
83.95	7.97
84.88	7.25
85.81	6.64
86.73	5.97
87.63	5.04
88.53	4.21
89.42	3.70
90.30	$3.36(10^{-6})$
91.17	3.06
92.03	2.82
92.88	2.66
93.72	2.49
94.55	2.29
95.37	2.10
96.18	1.89
96.98	1.72
97.78	1.54
98.56	1.39
99.33	1.28
100.10	1.19
100.85	1.12
101.60	1.05
102.33	$9.89(10^{-7})$
103.06	9.22

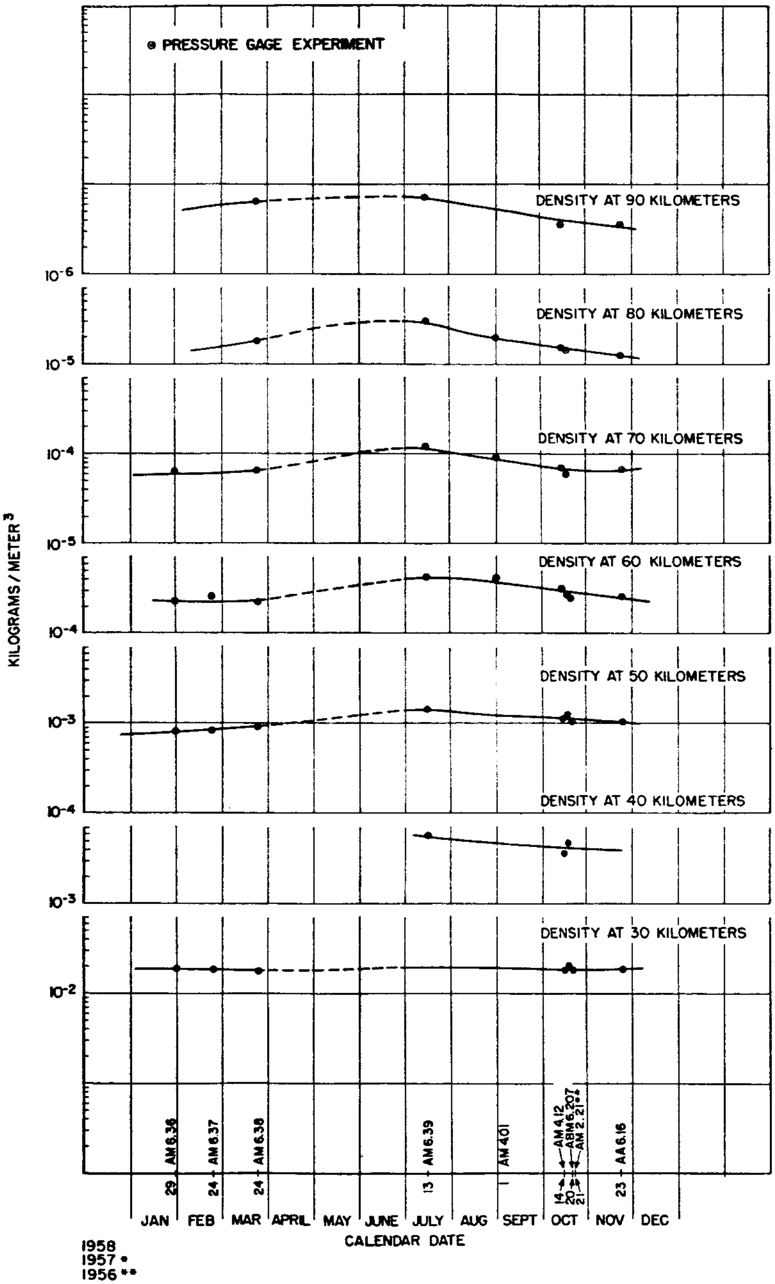

Fig. 3. Composite plot of selected density values obtained from pressure gage flights.

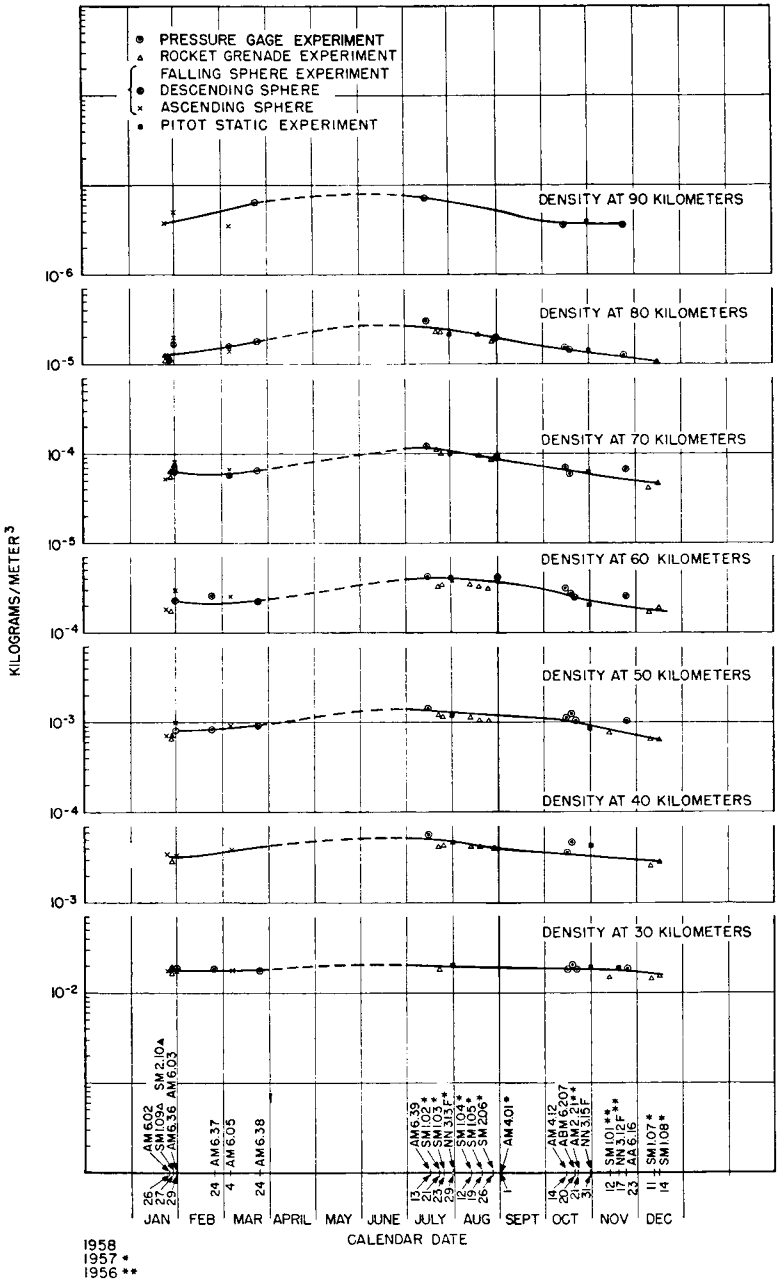

Fig. 4. Composite plot of selected density values obtained from Churchill flights.

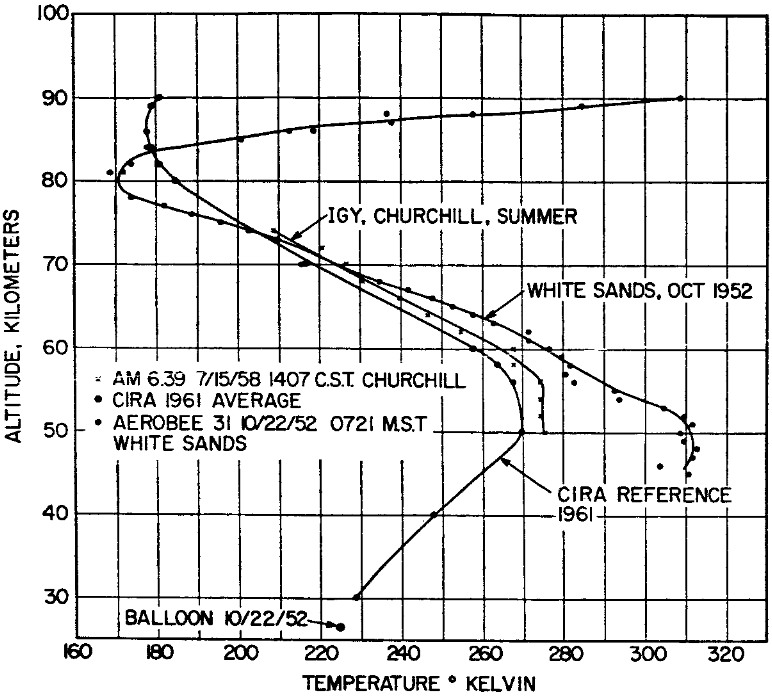

Fig. 5. Churchill summer data.

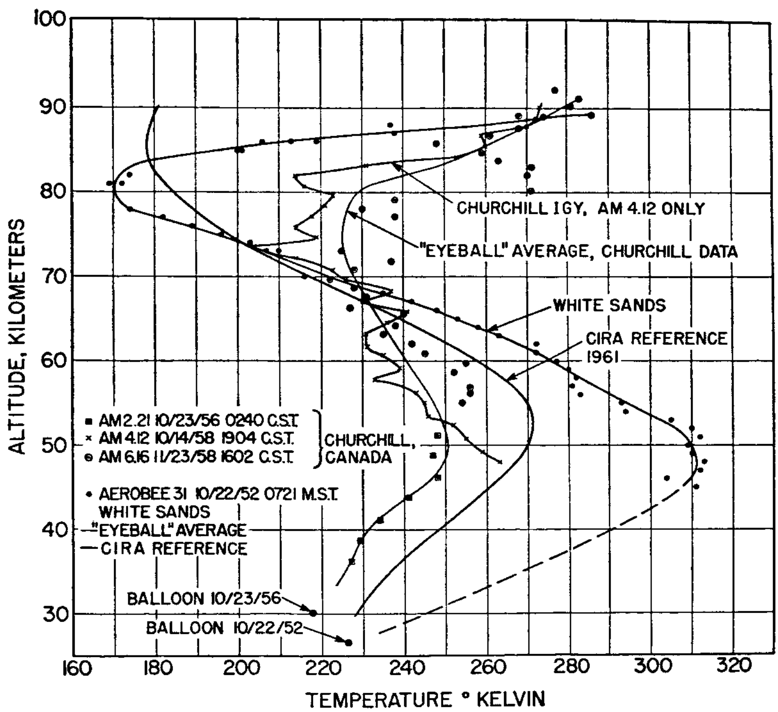

Fig. 6. Churchill winter data.

is significant. Careful attention has been given to a review of all gage calibrations to verify and establish their precision. It is to be noted further that usually the angles of attack were not high enough to perturb the ram-pressure data, as this measurement is relatively insensitive to angle of attack.

In summary, only data obtained using the Aerobee rocket was considered adequate to enable application of the Mach-number technique, as all Nike-Cajun flights experienced angles of attack greater than the nose-cone half-angle, thus rendering cone wall pressure data incapable of adequate analysis. On the other hand, use of Nike-Cajun ram-pressure data was considered satisfactory.

Density data interpretation. The density data shown in composite form in Figure 3 clearly demonstrate the seasonal variation of density in the subarctic region of Churchill, particularly at the higher altitudes. From the data obtained, it is concluded that the summer density in the 50- to 90-km region increases by a factor of 2 to 3 over the winter density during the period of solar maximum. Below 50 km, the increase is less, and in the region of 30 km is not detected. The absence of data durıng April, May, and June (launching facilities were unavailable during that period) does not appear to detract from these conclusions.

Temperature data interpretation. Comparison of the limited summer data (Figure 5) with the winter data (Figure 6) shows a strong seasonal change of temperature in the mesosphere and confirms the results of the grenade experiments published previously [*Stroud et al.*, 1960]. Particularly notable is the relative smoothness of the summer profile and the rather wide variations demonstrated by the winter data, as illustrated by the curve labeled AM 4.12. The data from AM 6.16 (circled dots, Figure 5) show the same local gradient structure as AM 4.12, but, in addition, show a strong secondary maximum around 82 km, which represents a departure from the relatively smooth profile usually observed previously at White Sands, and possibly lower latitudes.

The local variations observed in the AM 4.12 and 6.16 data are believed to be real. Although this point is difficult to establish firmly, the character of the experiment and the general lack of significant scatter in the telemetered data lead us to this conclusion. No correlation is observed between the instruments' pressure measurement resolution, which is somewhat periodic with altitude due to multiple ranging of the amplifier, and the observed temperature variability, as should be expected if the local variation were a consequence of scatter. The relative smoothness of the 'summer' data, wherein the same experiments were conducted in the same manner, also supports this argument. We consider these variations indicative of the dynamic state of the Churchill atmosphere in the winter, in contrast with the more nearly equilibrium conditions of the summer atmosphere. The relatively great altitude resolution capability of the experiments reported here permit examination of such variation, as was somewhat evident in the grenade results [*Stroud et al.*, 1960], for example.

The magnitude of the temperature maximum at 50 km in the White Sands data seems great in comparison with the reference atmosphere values. The data from this flight were derived employing the Mach-number technique under the most favorable conditions, particularly in the 50-km region where the rocket generally experiences very small angles of attack and the pressure gages' calibrations are most satisfactorily established. The only significant uncertainty for the flight was in the trajectory, and hence velocity data, which can effect the temperature values (see following section). The velocity data employed were derived from radar and were checked by comparison with data derived from an assumed vacuum trajectory based on the observed rocket peak altitude time. Use of velocity data derived in this manner reduces the maximum temperature by about 5°, resulting in a maximum some 10° higher than a value credited in the 1962 U. S. Standard Atmospheres as being an observed extreme. However, in summary, the Figure 6 profile resulted from due consideration of the experimental data and thus is supported by us as the most probable temperature of the atmosphere at that time.

Error estimate. The combined errors in data reduction and gage calibration produce an estimated over-all maximum error in raw pressure data of about 5 per cent. Since each point analyzed is independent of previous data read, the random errors, not dependent on systematic calibration errors, can be largely eliminated by smoothing processes, care being taken not to suppress the fine structure of the pressure profile.

The effect of this error in pressure data on determination of Mach number and temperature can be approximated to the first order by viewing the relationships involved in a plot of P_i/P_a (impact versus ambient pressure) versus Mach number, which is approximately linear in the region where the Mach number is larger than 3 but less than 5. Thus, in this case the error in the pressure ratio is, to the first order, directly reflected in the determination of Mach number.

By using the well-known definition of Mach number, we see that

$$M^2 = V^2/\gamma RT$$

Assuming that the error in velocity is zero, we can reduce this to

$$\Delta T/T = -2\,\Delta M/M$$

The error in temperature derived from Mach number is thus twice the error in Mach number.

This magnification of the error could present a problem in producing a good temperature profile. However, since the pressure ratio P_i/P_a is the quantity used for Mach number and temperature calculations, any system errors in gage calibration can be eliminated from the ratio if all gage units are calibrated simultaneously. This practice was followed.

For the alternate method of direct density determination from ram pressure, where the density is determined from a linear function of pressure, the error in pressure is directly reflected as an error in density.

Concluding remarks. We consider the observed seasonal variation of ambient density at high latitudes and during a period of solar maximum to be a positive and significant result of the experimental program. Although it is considered unfortunate that similarly extensive temperature data could not be presented with confidence, the fine detail evident in those data shown in Figure 6 is believed to be typical of Churchill profiles during that period.

Appendix

Theoretical aspects of cone surface-pressure measurements: relationship between total head and cone wall pressure. The considerable theoretical as well as experimental investigations of the flow properties of a right circular cone that have been carried out have facilitated the use of this configuration for the determination of the free stream Mach number and static pressure by direct measurement of the total head pressure (P_i) and cone wall pressure (P_c). (Symbols are defined at the end of the appendix.)

As has been shown [*Sicinski et al.*, 1954]

$$\frac{P_i}{P_c} = \frac{P_i}{P_a} \times \frac{P_a}{P_w} \times \frac{P_w}{P_s} \times \frac{P_s}{P_c} \tag{1}$$

For isentropic flow of a perfect gas, with adiabatic flow assumed across shocks, the ratios on the right-hand side of this identity are as follows:

Rayleigh pitot tube equation:

$$\frac{P_i}{P_a} = \left(\frac{\gamma + 1}{2} M^2\right)^{\gamma/(\gamma-1)} \cdot \left(\frac{2\gamma}{\gamma + 1} M^2 - \frac{\gamma - 1}{\gamma + 1}\right)^{1/(\gamma-1)} \tag{2}$$

Taylor-Maccoll cone theory as tabulated by *Kopal* [1947]:

$$\frac{P_w}{P_a} = \frac{(\gamma^2 - 1)(C^2 - U_w^2 - V_w^2)}{4\gamma V_w^2 - (\gamma - 1)^2(C^2 - U_w^2 - V_w^2)} \tag{3}$$

Bernoulli integral assuming isentropic flow of perfect gas behind shock:

$$\frac{P_w}{P_s} = 1 - \left(\frac{V_w}{C}\right)^2 - \left(\frac{U_w}{C}\right)^{2\gamma/(\gamma-1)} \tag{4}$$

By letting V_w approach zero in (4), we get

$$\frac{P_c}{P_s} = 1 - \left(\frac{U_w}{C}\right)^{2\gamma/(\gamma-1)} \tag{5}$$

Substituting (2) through (5) into the ratio identity (1) yields P_i/P_c as a function of M only. When we know the Mach number and the equation

$$M = V/a = V/\sqrt{\gamma RT} \tag{6}$$

it follows that

$$T_a = (1/R\gamma)(V/M)^2 \tag{7}$$

Therefore, the measurements required to determine ambient pressure P_a and temperature

T_a are the impact or ram pressure P_i, the cone wall pressure P_c, and the rocket velocity V. The equation of state ($P_a = \rho R T_a$) will yield the ambient density directly from P_a and T_a.

The above theoretical analysis assumes that the angle of attack of the cone is zero. For angles of attack other than zero, the equations must be altered somewhat to describe the flow. There have been many theoretical considerations of this problem, but no exact solution is available to date. However, the *Ferri* [1951] approach to the problem agrees with experimental data to a better degree than most other theoretical considerations.

For the experiments discussed in this paper employing the Nike-Cajun vehicles, the yaw angles for which the theory is valid were greatly exceeded. When the angles are larger than the half-angle of the cone, errors in Ferri's theoretical values as compared with experimental (wind tunnel) evidence can be as large as 80 per cent for values attained. Errors of this order are, of course, too large for data reduction by the process discussed to be worth while.

Impact pressure relationships. It has been noted that the impact pressure is unaffected by yaw for angles of attack as great as 30° for selected inlet ports, which is naturally a very attractive feature of an impact pressure measurement. The form of the Rayleigh equation which is useful in illustrating the significant aspects of the impact pressure measurement is:

$$\frac{P_i}{P_a} = \frac{\gamma + 1}{2} M^2 \left(\frac{(\gamma + 1)^2 M^2}{4\gamma M^2 - 2\gamma + 2} \right)^{1/(\gamma - 1)} \quad (8)$$

For $\gamma = 1.4$ (air) and for Mach numbers for the approximate value shown, this expression can be simplified for practical purposes to:

$$P_i/P_a = 1.326 M^2 \quad \text{for} \quad M \cong 3.5$$
$$P_i/P_a = 1.317 M^2 \quad \text{for} \quad M \cong 4.0 \quad (9)$$
$$P_i/P_a = 1.29 M^2 \quad \text{for} \quad M \to \infty$$

With the Aerobee and Nike-Cajun vehicles used, and in the ranges of altitude (30 to 90 km) in which data are analyzed, the Mach number varies between 3.5 and 5.5. Therefore, if we assume $P_i/P_a = 1.30 M^2$, the error will not be larger than 1 per cent for this altitude and Mach-number range.

Using the equation of state and the definition of M (equation 6), we obtain

$$P_i/P_a = 1.30 \rho V^2 / \gamma P_a \quad (10)$$

The ambient pressure terms cancel, and again using $\gamma = 1.4$ we find that the ambient density as a function of impact pressure and velocity becomes

$$\rho = 1.5 P_i / V^2 \quad (11)$$

where the units are:

ρ, kilograms per meter cubed.
P, millimeters of mercury.
V, feet per second.

Assuming an atmosphere in equilibrium, the hydrostatic equation, $dP_a/dh = \rho g$, can be used, yielding P_a:

$$P_{a(h_1)} = \int_{h_1}^{h_2} \rho g \, dh + P_{a(h_2)}$$

By assuming a temperature at h_2, say the 1959 ARDC value, and knowing the density at h_2, we can compute $P_{a(h_2)}$ by using the equation of state. The value of g is chosen as the average between the value of h_1 and h_2. The so-called correction value of $P_{a(h_2)}$ will become negligible when the integration is removed from h_2 by more than 15 km; therefore even large errors in $P_{a(h_2)}$ will not introduce errors in $P_{a(h_1)}$.

After we find $P_{a(h_1)}$, the temperature is readily computed from the equation of state:

$$T = P_a / \rho R$$

Symbols

- a, velocity of sound $= \sqrt{(\gamma R T)}$.
- c, limiting velocity, attained only theoretically by considering all heat energy converted into uniform motion.
- h, altitude.
- M, free stream Mach number $= V/a$.
- P, pressure, always followed by clarifying subscript.
- R, gas constant for air. Molecular weight $=$ 28.97 (sea level).
- T, temperature.
- U_w, Taylor-Maccoll velocity component along ray.
- V_w, Taylor-Maccoll velocity component perpendicular to ray.

V, velocity of body relative to ambient air.
ρ, ambient density.
γ, ratio of specific heats = 1.40 for air.

Subscripts

a, ambient values relative to ambient air.
c, cone surface values.
i, impact values.
s, stagnation values behind shock.
w, static values immediately behind shock wave.

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